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Cultivating Primary School Students' Inductive Thinking through the "Problem Chain" Approach: Taking the Example of "The Sum of Interior Angles of Polygons" in the People's Education Press Primary School Mathematics Textbook

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Abstract

Primary school mathematics serves as a crucial cornerstone of basic education and plays a vital role in developing students' logical thinking and other essential skills. Inductive thinking is a necessary competence for students, helping them to discover patterns, summarize methods, and transfer and apply knowledge. However, the cultivation of inductive thinking has not received sufficient attention in current primary mathematics teaching, and teaching methods need further optimization. The problem chain teaching method, which uses a series of interconnected questions to guide learning, can effectively stimulate students' thinking vitality. Integrating this method with the cultivation of inductive thinking can inject new energy into classroom teaching, enhancing both students' inductive thinking abilities and their overall mathematical literacy. This study aims to explore how to cultivate primary school students' inductive thinking through problem chain teaching, providing valuable insights for future teaching reform.

Keywords: problem chain; primary school mathematics; inductive thinking.

1. Introduction to the Problem

Inductive thinking is one of the fundamental methods of mathematical thinking, with rich and unique applications in both mathematical research and mathematical learning. Gauss once said that the use of inductive thinking can lead to the discovery of beautiful new truths. [1] The Compulsory Education Mathematics Curriculum Standards (2022 Edition) clearly states that mathematics courses should cultivate students' core competencies, among which inductive thinking, as an important way of thinking, runs through all stages of mathematical learning. It helps students discover patterns and summarize methods from concrete mathematical examples, thereby enabling the transfer and application of knowledge. This lays a solid foundation for students' future mathematical learning and lifelong learning. [2]

Problem chain teaching, as an innovative and effective teaching method, has gained considerable attention in the education field in recent years. It relies on a series of logically connected, progressively deepening questions that guide students to actively explore knowledge and deeply reflect on mathematical principles while solving problems. In primary school mathematics classrooms, the appropriate use of problem chains can spark students' interest in learning, encourage active classroom participation, and present abstract mathematical knowledge in a question-based, step-by-step manner. This approach simplifies complex concepts and helps students gradually build their thinking skills, step by step enhancing their overall cognitive ability.

The organic integration of problem chain teaching with the cultivation of inductive thinking offers multiple benefits. On the one hand, problem chains provide rich materials and pathways for developing inductive thinking. Through carefully designed questions, students are guided to observe, analyze, and compare, gradually accumulating perceptual knowledge that eventually rises to rational induction. On the other hand, the cultivation of inductive thinking further supports students in better solving problems within the problem chain. This allows them to generalize from specific cases, flexibly apply learned knowledge to various mathematical problems, improve the quality and efficiency of their mathematical learning, and promote the overall enhancement of their mathematical literacy.

In primary school mathematics teaching, traditional teaching methods often fail to help students develop deep understanding. Here's an example from traditional teaching practice:
The teacher writes multiplication equations on the blackboard and directly explains the distributive property of multiplication, then demonstrates some example problems for students to calculate. When practicing, some students can apply the formula correctly, but they struggle with variations, leading to errors. After repeated explanations by the teacher, the results are still unsatisfactory, and homework feedback shows a high error rate. The main issue is that students cannot flexibly apply the distributive property and are limited to mechanical formula memorization. For example, when solving problems like $99\times23+23$ or $38\times101-38$, students cannot effectively apply transformation strategies, instead following the usual step-by-step multiplication, which results in cumbersome calculations and frequent errors.

This case exposes several issues with traditional teaching methods. First, teachers focus too much on direct knowledge transmission, neglecting the process of student inquiry and exploration. Simply presenting and explaining formulas, followed by requiring students to memorize the distributive property formula, deprives them of the opportunity to deeply think about why the property holds. As a result, students fail to truly understand the essence of the distributive property and can only apply it mechanically, limiting their ability to use it flexibly.

Second, the teaching process lacks effective thinking guidance. The teacher does not introduce a sequence of progressive questions to guide students to observe, compare, and analyze relationships between equations. This absence of thought-provoking questions prevents students from discovering patterns independently, thus hindering their ability to practice and develop inductive thinking.

Furthermore, students' learning autonomy is not fully activated. The entire lesson is dominated by teacher explanations, with students passively receiving knowledge. They lack opportunities for independent exploration and collaborative discussion, resulting in low engagement and weak knowledge retention. These problems highlight the shortcomings of traditional teaching methods in fostering students' inductive thinking, demonstrating the urgent need for innovative approaches such as problem chain teaching. By incorporating problem chain teaching, students can be actively guided to participate in knowledge construction and gradually develop their inductive thinking through exploration and discovery.

2. Theoretical Analysis of Problem Chains and Inductive Thinking

(1) Exploration of Problem Chains

1. The Concept of Problem Chains

The renowned mathematician Paul Halmos once pointed out that "problems are the heart of mathematics" [3]. Some scholars also emphasize that "problems are the heart of teaching for teachers and the heart of learning for students," and that "human development promotes problem-solving, while problem-solving guides human development" [4]. This highlights the key role problems play, whether in fostering human development or advancing the discipline of mathematics itself.

Doubt arises from questions, thinking is triggered by doubt, and development is driven by thinking. To enable students to gradually deepen their thinking and foster development through problems, the construction of problem chains is indispensable. Scholars hold different views on the definition of problem chains. Some define a problem chain as a sequential set of core problems presented to students during lessons [5]. Others believe a problem chain is a central thread (or a primary line with several branch problems) designed to align with the core teaching objectives and students' learning conditions, characterized by integration, inquiry, and progressive difficulty [6].

From this, a problem chain in primary school mathematics can be defined as a collection of core and branch problems presented in class, aligned with core teaching goals and tailored to students' learning levels. It possesses the features of integration, inquiry-driven exploration, and progressive cognitive development.

2. Types of Problem Chains

Scholars have categorized problem chains from various perspectives. From the perspective of teaching functions, Wang Houxiong (2010) classifies problem chains into seven types: introductory, differential, diagnostic, exploratory, transferable, flexible, and summarizing [7]. This study focuses on three types from Wang's classification: introductory problem chains, transferable problem chains, and summarizing problem chains.

Introductory problem chains are carefully designed to introduce the topic, ensure smooth transitions between topics, or set the stage for follow-up lessons. They also aim to capture students' attention and spark their curiosity and desire for knowledge. To help all students personally experience the process of deriving concepts, principles, and patterns, teachers must guide them back to their pre-existing cognitive frameworks. By connecting new content to familiar prior knowledge, students can integrate old and new knowledge within their zone of proximal development. They are also encouraged to use analogy, deduction, induction, and other methods to uncover the relationships between old and new knowledge [8].

Transferable problem chains refer to chains in which one problem naturally leads to the solution of other important problems, either horizontally or vertically. This type of chain aims to foster and consolidate new concepts, principles, methods, and rules, ultimately improving students' ability to apply these concepts flexibly in different contexts. Through these experiences, students personally witness the wide applicability of the knowledge they learn.

Summarizing problem chains are designed for concluding lessons or units, aimed at recalling and organizing previously learned knowledge into systematic structures. Their purpose is to guide students to independently summarize the structure and internal connections of the knowledge learned in a lesson or unit. Through carefully designed summarizing questions, scattered and isolated knowledge is consciously organized into an interconnected whole, forming a systematic and structured knowledge network. This fosters students' ability to summarize and organize knowledge effectively.

(2) The Concept of Inductive Thinking

Inductive thinking is a cognitive process that derives general conclusions by analyzing a set of specific cases within a certain category. Its epistemological foundation lies in the commonalities and similarities embedded in individual instances of the same kind of phenomena [9].

Inductive thinking has several characteristics:

Inductive thinking incorporates practical experience into the thinking process. It relies on various perceptual, concrete materials as its foundation and requires the generalization of rules from specific and individual cases.

The conclusions drawn through inductive thinking are probabilistic rather than absolute. To become reliable, these conclusions need to undergo rigorous verification and practical testing.

Inductive thinking serves as a bridge for individuals to ascend from perceptual understanding to rational understanding. On the basis of perceptual cognition, individuals acquire new concepts, judgments, and theories through inductive processes [10].

How Problem Chains Facilitate the Development of Inductive Thinking

Problem chains provide a powerful platform for cultivating inductive thinking.

First, problem chains present a carefully structured series of problems that guide students to purposefully observe and analyze mathematical phenomena and examples.

Second, they provide clear investigative pathways. When faced with complex mathematical concepts, students often struggle with where to begin. Problem chains break down and simplify knowledge, pointing students in the right direction for their thinking processes.

Third, the progressive thinking environment created by problem chains aligns with students' cognitive development stages, gradually advancing their inductive thinking abilities. Starting from simple, intuitive questions and gradually progressing to more abstract and complex ones, problem chains allow students to climb cognitive "steps" one by one.

Under the guidance of such a progressive sequence of problems, students' inductive thinking develops from superficial to deep, from concrete to abstract, achieving the leap from direct perception to abstract generalization.

3. Strategies for Cultivating Inductive Thinking Through Problem Chains

Cultivating students' inductive thinking is a crucial educational objective, and the skillful use of problem chains provides an effective pathway to achieve this goal. The following discussion uses the teaching of "Sum of Interior Angles of Polygons" from the People's Education Press (PEP) curriculum as an example to explore how carefully designed problem chains can foster students' inductive thinking from multiple dimensions.

(1) Designing Introductory Problem Chains to Spark Inductive Motivation

1. Creating Contextual Problems

At the beginning of the lesson on "Sum of Interior Angles of Polygons," the teacher creates a vivid and engaging contextual problem closely related to students' daily lives:
“Class, there are many beautiful polygon-shaped flower beds on our school campus. The school plans to renovate these flower beds with decorative tiles. To accurately estimate the materials needed, we first need to know the sum of the interior angles of these polygonal flower beds. But we can’t just dismantle each corner to measure them. Can you think of a good way to find out?”
This contextual problem immediately connects abstract mathematical knowledge with familiar campus scenes. Students visualize these familiar flower beds in their minds, which sparks strong curiosity about how to solve the problem of measuring the sum of interior angles in polygons. Their thinking is quickly activated, and they become eager to find solutions, laying a solid emotional and cognitive foundation for subsequent exploration.

2. Stimulating Inquiry Desire

Building on the above scenario, the teacher poses another question:
“We already know that the sum of the interior angles of a triangle is 180 degrees. Can we use this knowledge to find the sum of interior angles of polygons? Take a bold guess — do you think there’s a relationship between the interior angle sums of quadrilaterals, pentagons, hexagons, etc., and the interior angles of triangles?”
This question is like a pebble thrown into a calm lake, stirring up waves of student thinking. Drawing from their existing knowledge of triangle interior angles, students begin to actively think about how to find the sum of interior angles for polygons. They are filled with a strong desire to explore and are eager to uncover the mystery behind the sum of polygon interior angles. This inquiry impulse, triggered by the connection to prior knowledge, motivates students to fully engage in the subsequent learning process and actively observe and analyze the properties of polygon interior angles, injecting powerful momentum into the development of their inductive thinking.

(2) Constructing Transferable Problem Chains to Guide the Inductive Process

1. From Specific Cases to General Patterns

After igniting students' desire to explore, the teacher constructs a progressive problem chain to guide students into deeper inquiry. First, the teacher asks:
“Let’s start with quadrilaterals. Can you think of a way to use the triangle interior angle sum to find the sum of interior angles of a quadrilateral? Try it out — split the quadrilateral into triangles.”
Students begin hands-on exploration. Some draw diagonals to divide the quadrilateral into two triangles and quickly determine that the sum of interior angles is 360 degrees.

The teacher follows up:
"Great job! Now what about a pentagon? Try again — how many triangles can you split a pentagon into, and what's the sum of its interior angles?"
Students continue exploring and discover that a pentagon can be divided into three triangles, with a total interior angle sum of 540 degrees.
The teacher continues:
"How about a hexagon? How would you split it, and what's its interior angle sum?"
Through further attempts, students conclude that a hexagon can be split into four triangles, with an interior angle sum of 720 degrees.

Through the exploration of the interior angles of triangles, quadrilaterals, pentagons, and hexagons, students accumulate rich empirical understanding, laying a solid foundation for inducing the general rule for the interior angle sum of polygons.

The development of individual cognition can, to some extent, be seen as a miniature replay of humanity's collective cognitive development. This illustrates that mathematical learning is a simulated process — a "re-creation" process organized by the teacher. Students should become active participants rather than passive recipients of knowledge. They need opportunities to engage, to personally experience the process of knowledge discovery and development, and to go through the process of re-creating knowledge.

In this stage, the teacher guides students to divide polygons into triangles, gradually solving for the interior angle sums of polygons like pentagons and hexagons. This not only provides diverse materials for further inductive reasoning but also allows students to actively experience analogy and association, fostering logical reasoning skills and geometric intuition.

2. Gradually Deepening Inductive Reasoning

Once students develop a clear understanding of the interior angle sums of specific polygons, the teacher asks a deeper question:
"Class, look at the interior angle sums we've found for these polygons. Think carefully — how does the sum of the interior angles relate to the number of sides in the polygon? Discuss this in your groups and see if you can come up with a formula."
Students engage in lively group discussions, exchanging insights. Some groups discover that subtracting 2 from the number of sides and multiplying the result by 180 degrees gives the interior angle sum.
The teacher affirms their discovery and asks:
"Excellent thinking! Can we use the letter 'n' to represent the number of sides and write this formula more concisely?"
After further thought, students derive the general formula for the sum of interior angles of an n-sided polygon: $(n-2)\times180°$. At this point, the teacher poses one more question:

“Can this formula accurately find the interior angle sum of any polygon, no matter how many sides it has? Let’s test it with heptagons, octagons, and so on.”
Students substitute specific values for nnn and verify the formula. This hands-on validation process deepens their understanding and trust in the inductive result. Students not only master the formula for polygon interior angle sums but also enhance their inductive thinking through this gradual deepening process.

(3) Expanding a Summarizing Question Chain to Facilitate Inductive Communication

1. Group Inquiry

After students initially derive the formula for the sum of interior angles of a polygon, the teacher organizes group cooperative exploration. The teacher poses the question:
"Students, we just obtained the sum of interior angles of a polygon by connecting diagonals from a vertex. Are there any other methods of partitioning that can also be used to determine the sum of interior angles? Work in groups, draw and experiment, and see which group can discover a new method."

The students enthusiastically engage in discussion and hands-on exploration. Some groups attempt to select an arbitrary point inside the polygon and connect it to each vertex, dividing the polygon into multiple triangles. Others start from a point on the polygon’s edge and connect it to other vertices to create different partitions. Throughout the group inquiry process, students exchange ideas, inspire one another, and engage in critical thinking. Various partitioning strategies gradually emerge, with every student actively contributing their insights to discover new methods. This process further broadens and deepens their understanding of how to calculate the sum of interior angles of a polygon.

2. Sharing and Summarizing Findings

After the group inquiry, the teacher facilitates a session where each group shares its findings. A representative from each group presents their partitioning method using a projector and explains the calculation process for the sum of interior angles in detail. For instance, one group demonstrates a method in which they select a point inside the polygon and connect it to each vertex:

"We chose a point inside the polygon and connected it to all the vertices, dividing the polygon into as many triangles as its number of sides. However, an extra full rotation of 360° is created in the middle. Therefore, the sum of interior angles is calculated as $n\times180°-360°$, which simplifies to $(n-2)\times180°$."

Other groups listen attentively and compare different approaches. The teacher then guides

students to engage in comparative analysis and generalization:
"Look, although the partitioning methods are different, they all lead to the same formula for the sum of interior angles. What does this tell us?"

Through discussion and reflection, students gain a deeper appreciation of the diversity and consistency of mathematical methods, further refining their understanding of the sum of interior angles of a polygon. Meanwhile, during the sharing and communication process, students significantly improve their presentation skills, logical reasoning, and teamwork. The inductive thinking process is reinforced and elevated in an atmosphere of collaborative learning and exchange.

Summary

The problem chain, with its diverse types, weaves a tightly-structured cognitive guidance network for mathematics teaching. Closely aligned with teaching objectives and students' cognitive patterns, it breaks down knowledge into progressively layered and closely connected sets of questions, clarifying students' learning pathways. From the teaching example of "the sum of interior angles of polygons," it can be seen that introductory problem chains create contexts and stimulate inquiry, allowing students to transition naturally from real-life scenarios to mathematical thinking. Transfer problem chains follow the pattern of moving from the specific to the general, helping students gradually accumulate perceptual understanding and eventually summarize abstract mathematical rules. Concluding problem chains guide students in collaborative group discussions, where knowledge is consolidated and deepened through communication and exchange, expanding their thinking boundaries while enhancing their overall competencies.

Inductive thinking, as a key capability in elementary mathematics learning, evolves from simple observation and generalization in lower grades to more complex and abstract generalization in higher grades — a developmental process that aligns closely with problem chain teaching. Problem chains provide rich nourishment for the cultivation of inductive thinking, guiding students to systematically observe, deeply analyze, and accurately compare mathematical cases. Throughout the process of solving a series of related problems, students achieve cognitive leaps from the concrete to the abstract, from the specific to the general, successfully summarizing mathematical rules and methods, and ultimately internalizing and transferring knowledge.

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